# Nearly Perfect Fluidity in Cold Atomic Gases

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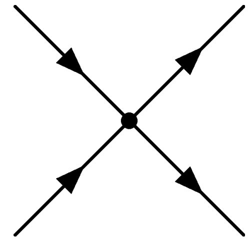
# Kinetic theory

High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$

Bruun (2005)



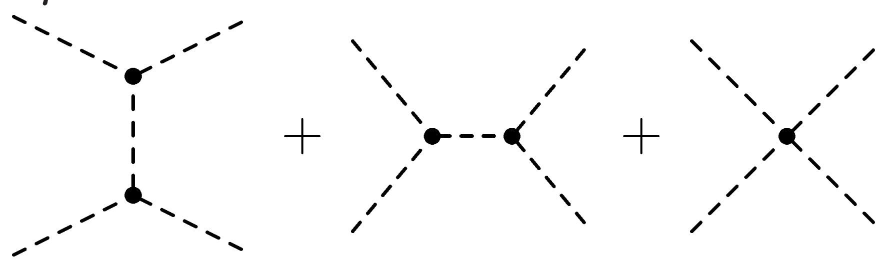
Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

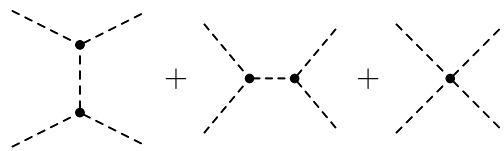
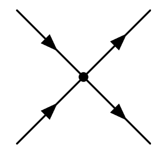
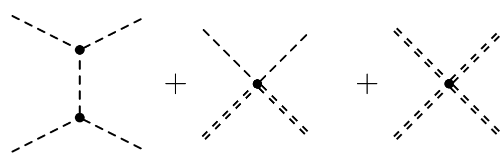
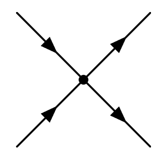
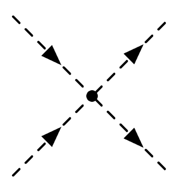
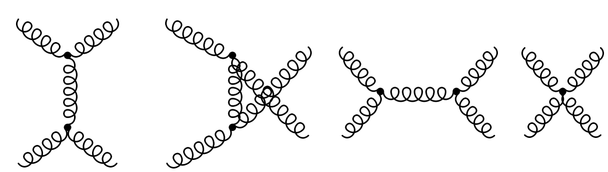
Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$

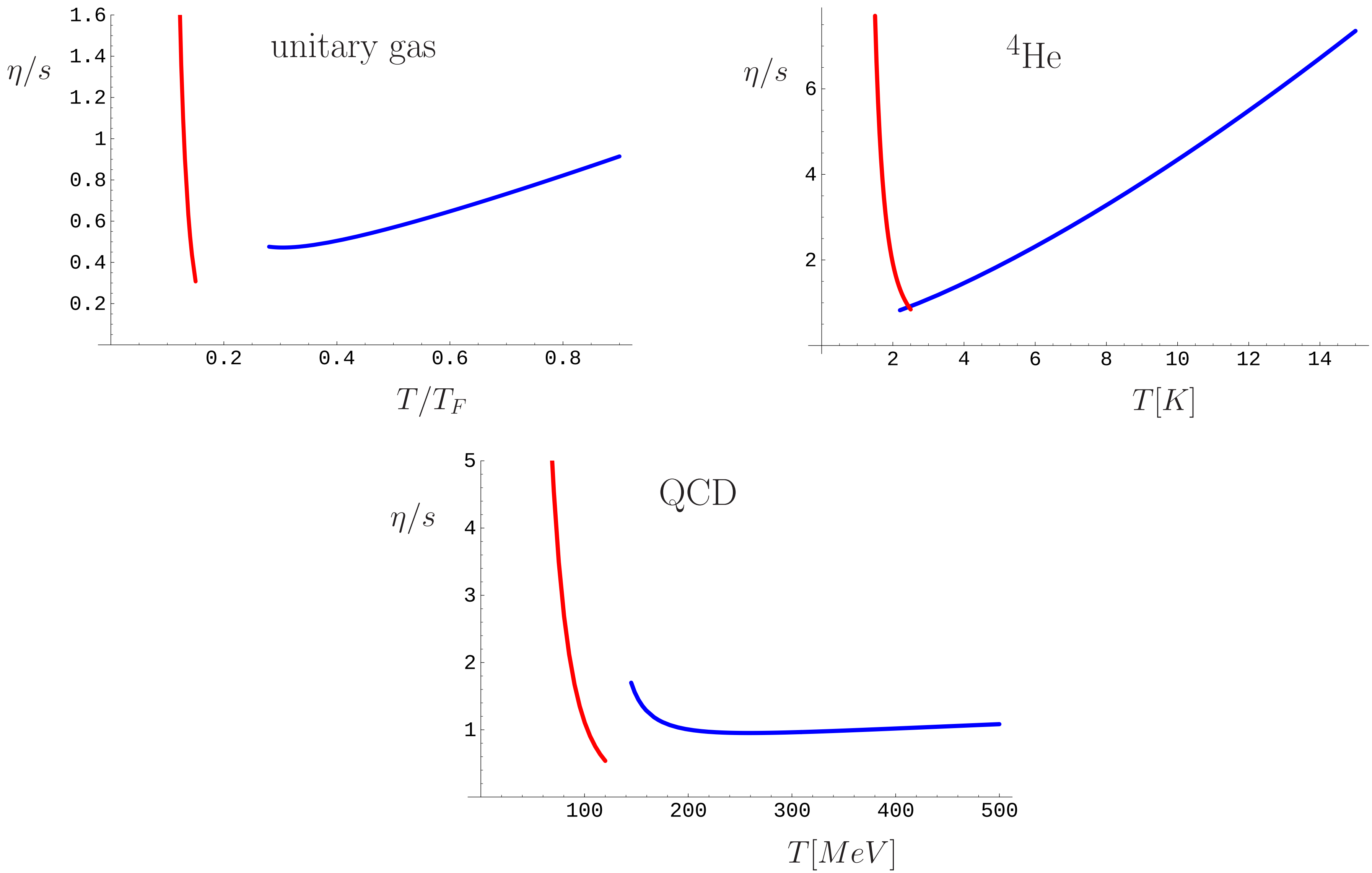
T.S., G.R. (2007)



# Kinetic Theory: Quasiparticles

| | low temperature | high temperature |
|---|---|---|
| unitary gas | phonons | atoms |
| |  |  |
| helium | phonons, rotons | atoms |
| |  |  |
| QCD | pions | quarks, gluons |
| |  |  |

# Kinetic theory summary



# Shear viscosity: Sum rules

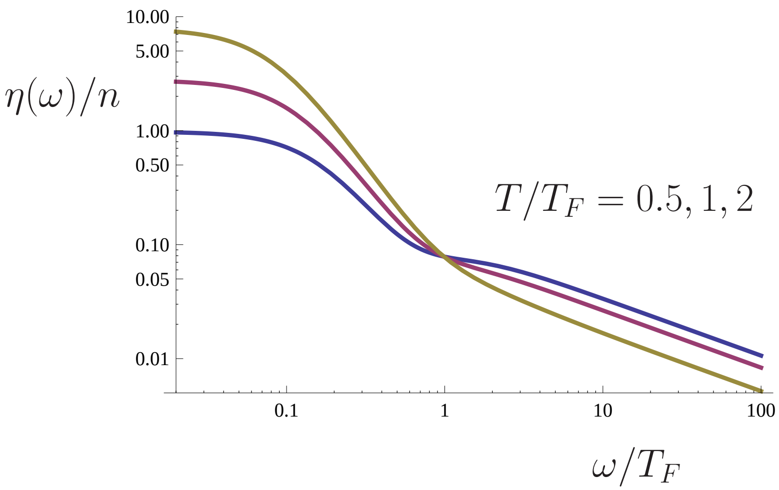
Randeira and Taylor proved the following sum rules

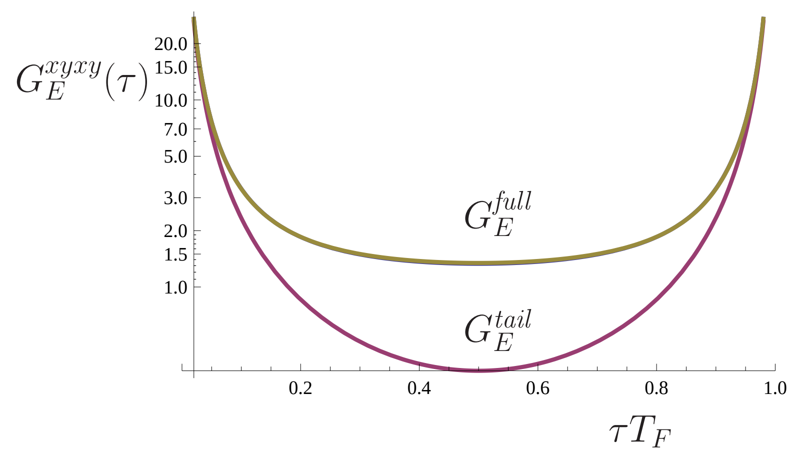
$$\frac{1}{\pi}\int dw\,\left[\eta(\omega)-\frac{C}{10\pi\sqrt{m\omega}}\right]=\frac{\mathcal{E}}{3}-\frac{C}{10\pi ma}$$

$$\frac{1}{\pi}\int dw\,\zeta(\omega)=\frac{1}{72\pi ma^2}\left(\frac{\partial C}{\partial a^{-1}}\right)$$

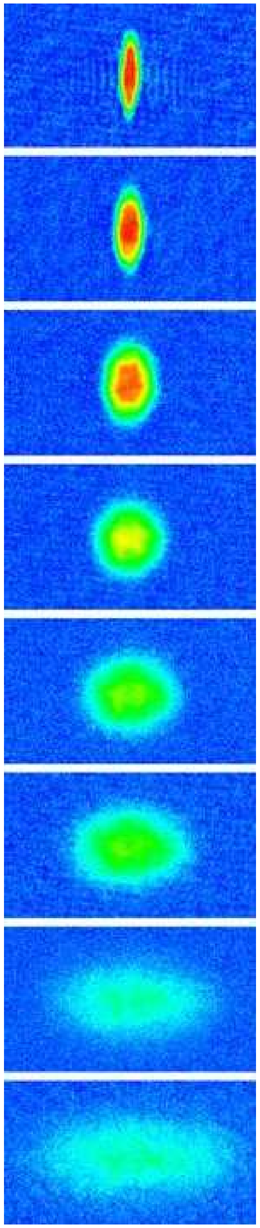
where $C$ is Tan's contact, $\rho(k)\sim C/k^4$.

## Sum rules constrain spectral function and euclidean correlator

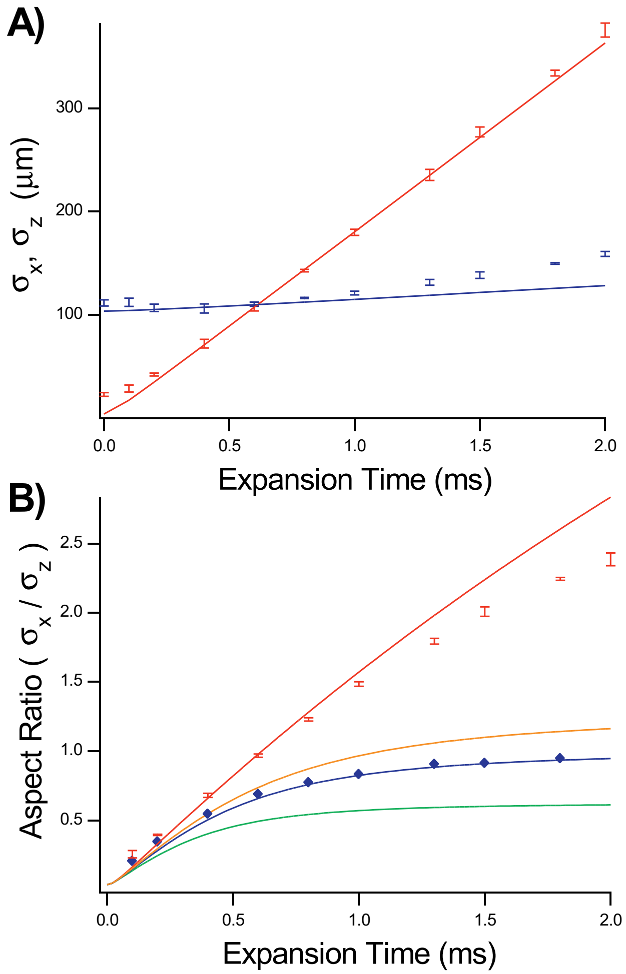


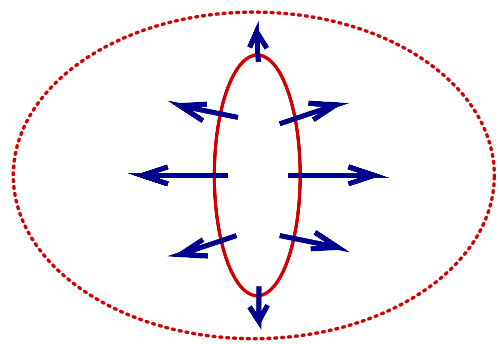
# Almost ideal fluid dynamics



O'Hara et al. (2002)

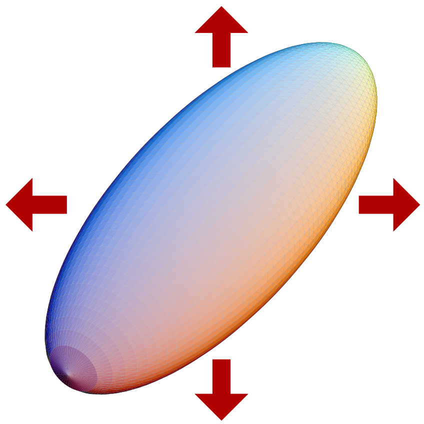


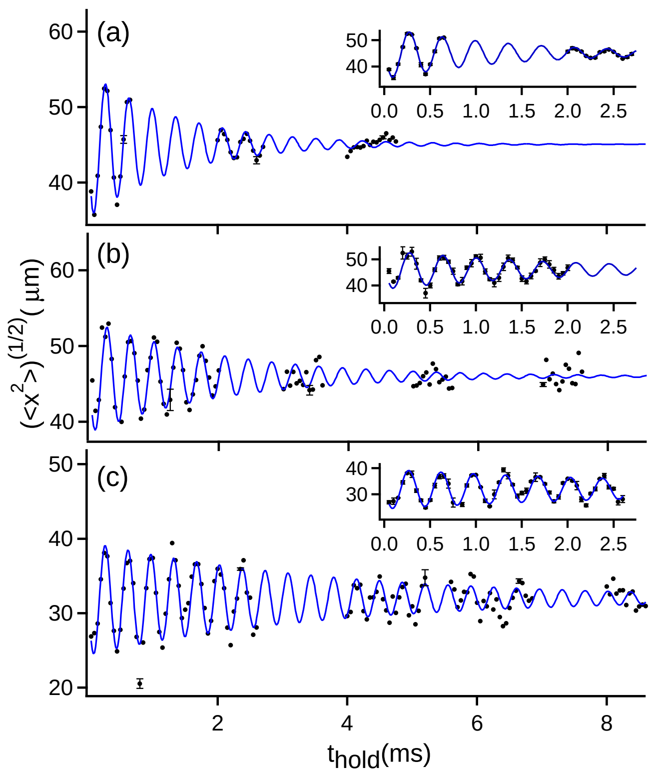

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



# Hydrodynamics: Collective modes

Radial breathing mode






Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}$$

Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\, \omega_\perp$$

Damping small, depends on $T/T_F$.

experiment: Kinast et al. (2005)

# Damping of collective mode

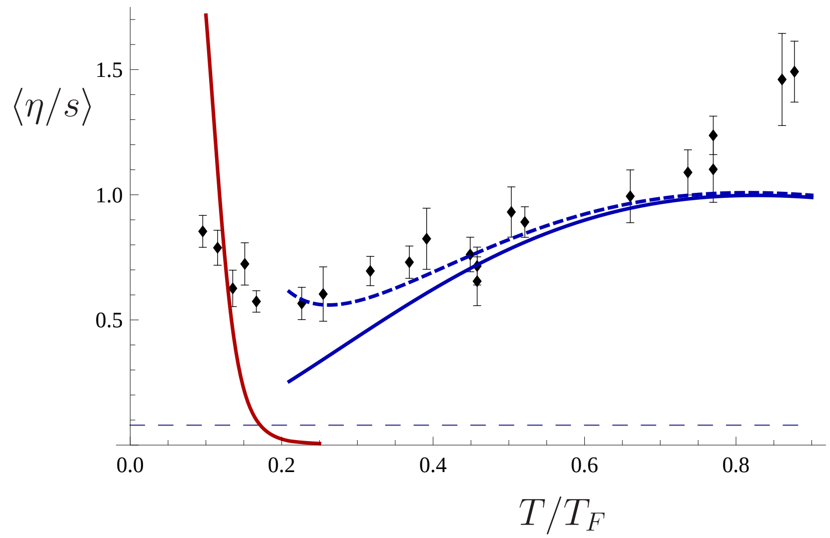
Energy dissipation ($\eta, \zeta, \kappa$: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

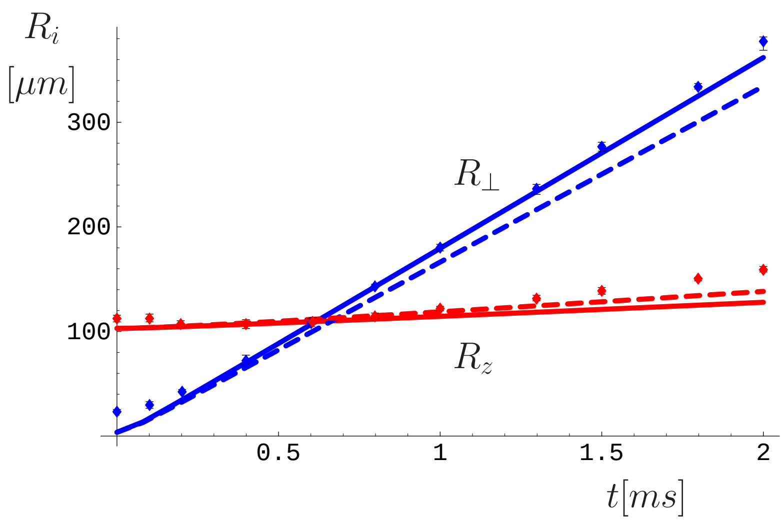
$$\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{E_0}{E_F}\frac{N}{S}$$

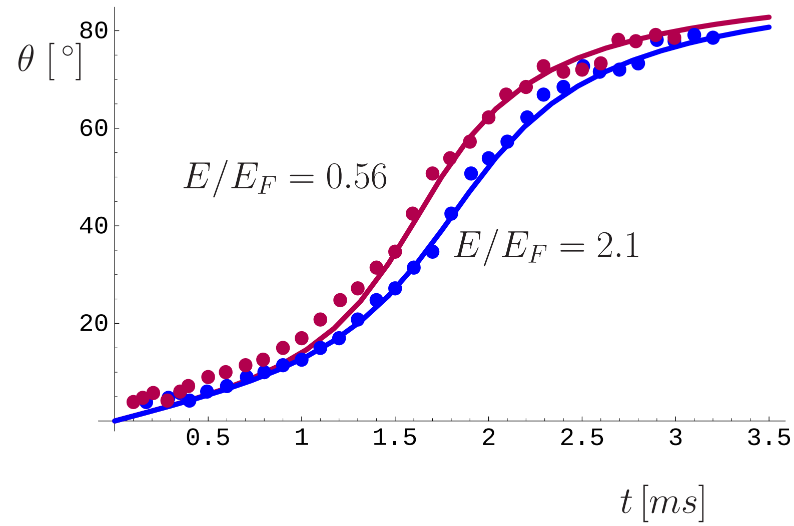


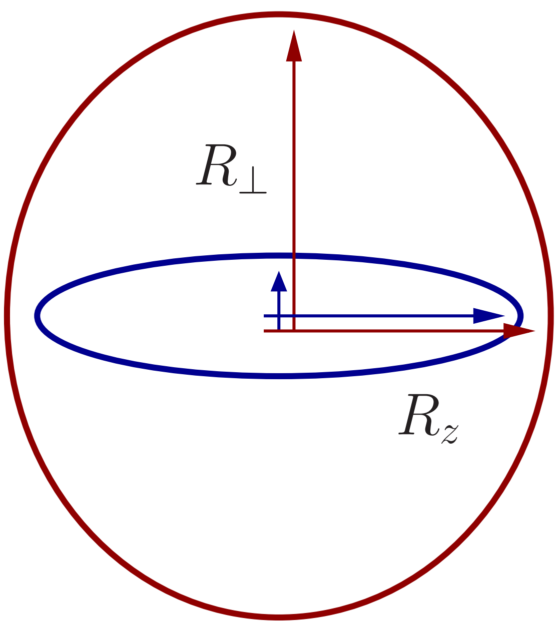

Schaefer (2007), see also Bruun, Smith

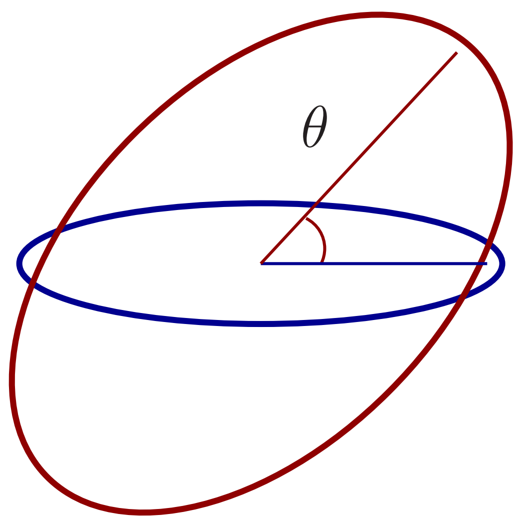
$T \ll T_F$ $\quad$ $T \gg T_F,\ \tau_R \simeq \eta/P$

# Hydrodynamics: Free expansion and rotation




O'Hara et al (2002), Clancy et al (2007)

# Scaling Flows

Universal equation of state $$P = \frac{n^{5/3}}{m} f\left(\frac{mT}{n^{2/3}}\right)$$

Equilibrium density profile

$$n_0(x) = n(\mu(x), T) \qquad \mu(x) = \mu_0 \left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

Scaling Flow: Stretch and rotate profile

$$\mu_0 \to \mu_0(t), \qquad T \to T_0(\mu_0(t)/\mu_0), \qquad R_x \to R_x(t), \ \ldots$$

Linear velocity profile

$$\vec{v}(x,t) = (\alpha_x x + (\alpha - \omega) y, \alpha_y y + (\alpha + \omega) y, \alpha_z z)$$

"Hubble flow"

# Scaling hydrodynamics

Write $R_i(t) = b_i(t)R_i(0)$. Euler equation

$$\ddot{b}_\perp = \frac{\omega_\perp^2}{(b_\perp^2 b_{||})^{2/3}}\frac{1}{b_\perp} \qquad b_\perp(\omega_\perp t \gg 1) \sim \sqrt{\frac{3}{2}}\,\omega_\perp t$$

Dissipation breaks scaling behavior. Consider moments of Navier-Stokes equation

$$\int d^3x\, x_k\,(\rho\dot{v}_i + \ldots) = \int d^3x\, x_k\,(-\nabla_i P - \nabla_j \delta\Pi_{ij})$$

Integration by parts: only sensitive to $\langle\eta\rangle/E_0$.

$$\ddot{b}_\perp = \frac{\omega_\perp^2}{(b_\perp^2 b_{||})^{2/3} b_\perp} - \frac{2\beta\omega_\perp}{b_\perp}\Big(\frac{\dot{b}_\perp}{b_\perp} - \frac{\dot{b}_{||}}{b_{||}}\Big)$$

with $\beta = \frac{\langle\eta\rangle}{N}\frac{E_F}{E_0}\frac{1}{(3N\lambda)^{1/3}}$

# Scaling hydrodynamics, continued

Friction term leads to delayed expansion

$$\frac{\delta t_0}{t_0} = 0.008\left(\frac{\langle\eta/s\rangle}{1/(4\pi)}\right)\left(\frac{2\cdot 10^5}{N}\right)^{1/3}\left(\frac{S/N}{2.3}\right)\left(\frac{0.85}{E_0/E_F}\right)$$

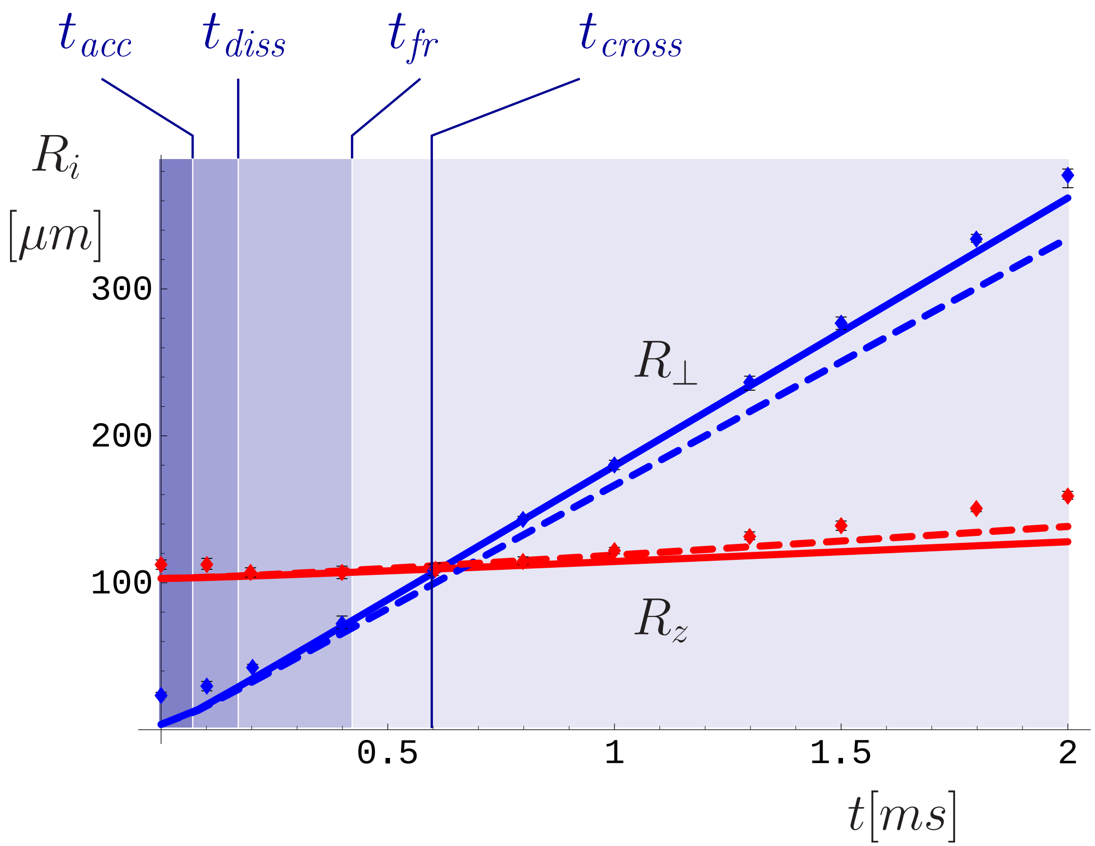
Effect does not exponentiate, harder to observe.

Issues: Reheating not taken into acount.

Dilute corona $\eta \sim T^{3/2} \to \nabla_i \delta\Pi_{ij} = 0$. No force (?)

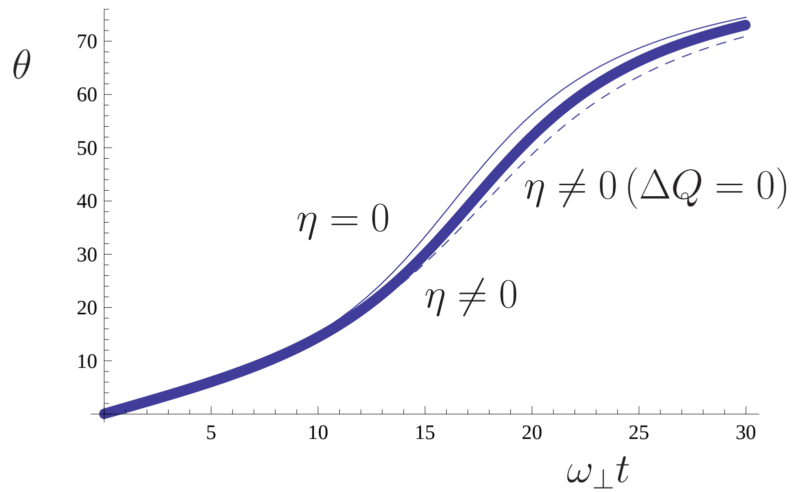
$Kn \sim (b_{||}/b_{\perp})^{1/3}$ drops $\to$ No freezeout (?)

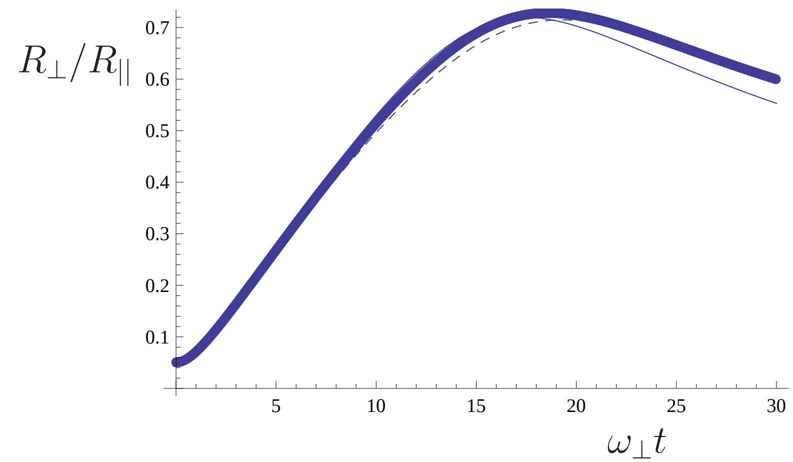
# Time Scales



# Navier-Stokes: Numerical results

Consider $\eta = \alpha_n n$. System parameters $\omega_z = 0.045\omega_\perp$, $\Omega = 0.4\omega_z$.




Reheating $\Delta Q = T\Delta S = \frac{\eta}{2}(\partial_i v_j + \ldots)^2$ counteracts viscous forces.

Scaling solution overestimates viscous effects by factor $\sim 2$.

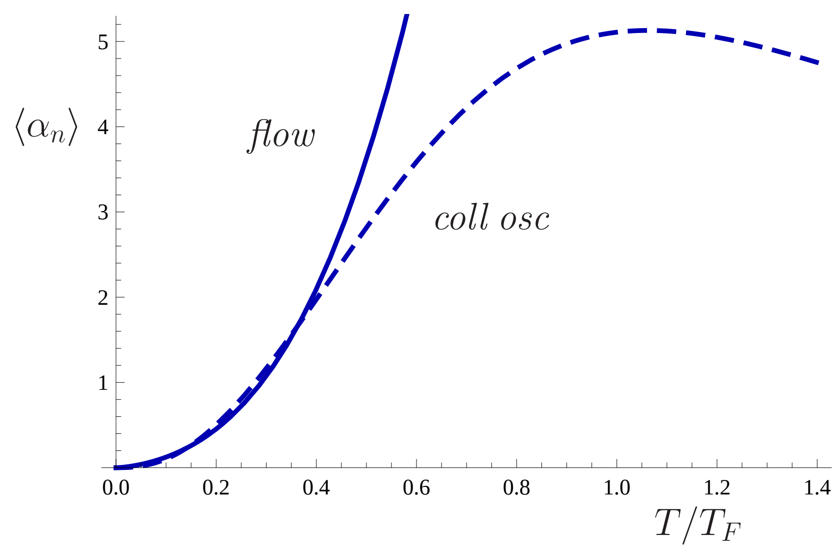
# Relaxation time model

In real systems stress tensor does not relax to Navier-Stokes form instantaneously. Consider

$$\tau_R\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\delta\Pi_{ij}=\delta\Pi_{ij}-\eta\left(\partial_i v_j+\partial_j v_i-\frac{2}{3}\delta_{ij}\partial\cdot v\right)$$

In kinetic theory $\tau_R \simeq (\eta/n)\,T^{-1}$

- disspiation from $\eta \sim (mT)^{3/2}$: corona excerts drag force.
- find $\langle\alpha_n\rangle \sim T^3$
- system dependence

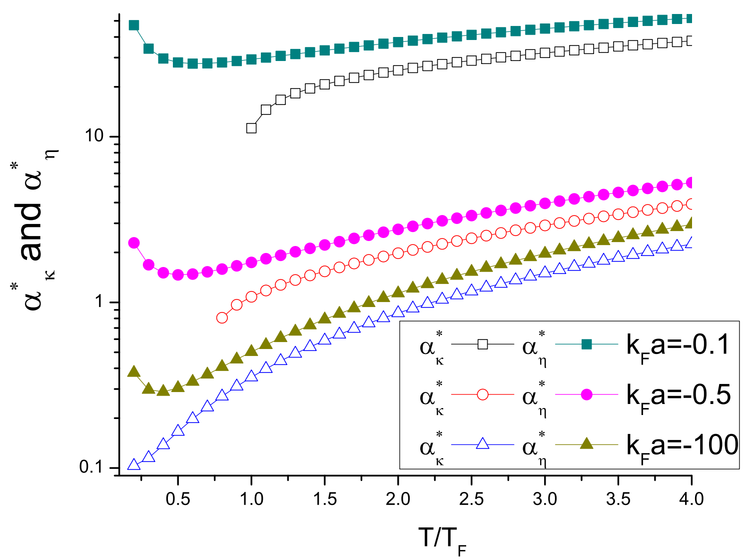


# Other ideas: Sound absorption

Sound absorption coefficient $\alpha = \gamma/\omega$ ($\delta n = \delta n_0 \exp(-2\gamma x)$)

$$\alpha = \frac{\gamma}{\omega^2} = \frac{1}{2\rho c^3}\left[\frac{4}{3}\eta + \rho\,\kappa\left(\frac{1}{c_V} - \frac{1}{c_P}\right)\right]$$

is directly sensitive to density independent part of $\eta$.




Kinetic theory preditcs

$$\kappa = \frac{225}{128\sqrt{\pi}} m^{\frac{1}{2}} T^{\frac{3}{2}} \quad Pr = 2/3$$

$T < T_F$: Sound absorption dominated by shear viscosity

Braby, Chao, T.S. (2010)

# Outlook

The unitary Fermi gas is an important model system for other strongly correlated quantum fluids in nature (the quark gluon plasma, dilute neutron matter)

The equation of state has been determined to a few percent.

Transport properties are more difficult: Kinetic theory at $T \gg T_F$ and $T \ll T_F$. Sum rules constrain spectral fct at all $T$.

Experimental determination of transport properties: Collective modes give $\eta/s < 0.5$. Analysis of expanding systems still in progress. Requires full second order hydrodynamics.